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## A framework to foster problem-solving in STEM and computing education

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### ABSTRACT

**Background:** Recent developments in STEM and computer science education put a strong emphasis on twenty-first-century skills, such as solving authentic problems. These skills typically transcend single disciplines. Thus, problem-solving must be seen as a multidisciplinary challenge, and the corresponding practices and processes need to be described using an integrated framework.

**Purpose:** We present a fine-grained, integrated, and interdisciplinary framework of problem-solving for education in STEM and computer science by cumulatively including ways of problem-solving from all of these domains. Thus, the framework serves as a tool box with a variety of options that are described by steps and processes for students to choose from. The framework can be used to develop competences in problem-solving.

**Sources of evidence:** The framework was developed on the basis of a literature review. We included all prominent ways of domain-specific problem-solving in STEM and computer science, consisting mainly of empirically orientated approaches, such as inquiry in science, and solely theory-orientated approaches, such as proofs in mathematics.

**Main argument:** Since there is an increasing demand for integrated STEM and computer science education when working on natural phenomena and authentic problems, a problem-solving framework exclusively covering the natural sciences or other single domains falls short.

**Conclusions:** Our framework can support both practice and research by providing a common background that relates the ways, steps, processes, and activities of problem-solving in the different domains to one single common reference. In doing so, it can support teachers in explaining the multiple ways in which science problems can be solved and in constructing problems that reflect these numerous ways. STEM and computer science educational research can use the framework to develop competences of problem-solving at a fine-grained level, to construct corresponding assessment tools, and to investigate under what conditions learning progressions can be achieved.

### KEYWORDS

Problem-solving; inquiry;  
STEM education; computer  
science education

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## Introduction

There has been a shift in science education to ‘focus from simply teaching science ideas to helping students figure out phenomena and design solutions to problems’ (Krajcik 2015, 16). This new view, also denoted as a *practice turn*, puts emphasis on creating an epistemic culture by letting students actively work in authentic scientific inquiry processes (Forman 2018); it is, thus, grounded in the image of science learning as *practice* (Lehrer and Schauble 2015) or as *participation* (Sfard 1998). Some of the corresponding competencies are described in the science standards of different countries, e.g. the NGSS in the United States (NGSS Lead States 2013), the National Curriculum in England (Department for Education 2015), or the Bildungsstandards für den Mittleren Schulabschluss in Germany (KMK 2004a, 2004b, 2004c). Acquiring scientific understanding through solving problems that are feasible, worthwhile, contextualized, meaningful, ethical, and sustainable (Krajcik 2015, 18) makes it necessary to integrate content, procedural, and epistemic knowledge (Kind and Osborne 2017). Furthermore, this multi-dimensional teaching approach often involves different domains like science, technology, engineering, mathematics (STEM),<sup>1</sup> and computer science simultaneously. Hence, problem-solving is an activity intrinsic to all of these domains, and, thus, it can serve as a general and common approach to teaching.

Enabling students to solve these kind of interdisciplinary and authentic problems calls for a general epistemological framework that supports problem-solving by providing a set of general scientific processes and methods that reflects the variety of different scientific approaches that might be needed. A problem-solving model exclusively covering the natural sciences or other single domains falls short because it limits its applicability when solving interdisciplinary problems or, as Kind and Osborne emphasize, ‘Mathematical and computational thinking cannot be excised’ (2017, 25).

Going further, positing that mathematical, computational, engineering, technological, and scientific thinking can and should not be seen as isolated, we developed a general framework that *does not* deliver a single method to solve a problem. Instead, it provides a set of idealized, principal problem-solving processes (epistemological ways) originating from the STEM domains that can be selected and possibly combined, adapted, and applied to a specific context. Thus, we provide a novel domain-general framework (detached from context) that works as a methodological ‘quarry’, where students choose bits and pieces and then customize and apply them according to their specific problem. We desire to make clear that it is not our intention to say that students should memorize procedures encoded in the framework and, thereby, gain a deeper understanding of scientific processes or acquire problem-solving skills. This would reflect the ‘old’ view that science practices merely follow rote laboratory procedures. Instead, it is the mastery of the use of the epistemological ways in multiple contexts that makes students, as members of a community of practice, literate in scientific problem-solving. Thus, we focus on epistemic knowledge ‘that should be the outcome of any education in the sciences’ (Kind and Osborne 2017, 24). What is new about our approach, moreover, is that our framework covers all STEM domains. While Osborne (2013), on the one hand, addresses science in detail, engineering is not mentioned explicitly, and mathematics is not mentioned at all. On the other hand, PISA (OECD 2013) focuses on problem-solving in science and mathematics and relates problem-solving in these domains, but it does not address engineering or computer science.

Thus, our framework can support practice by using epistemological ways to introduce students to methods and practices of problem-solving used in the STEM domains. Given a certain contextual problem, students can learn to identify, use, compare, and evaluate multiple scientific methods and, therein, gain insight into a wide spectrum of different ways to approach a problem. Furthermore, it can help students to understand that there is no *one* way to solve a problem, a view to which students struggle to adapt (Erduran and Dagher 2014). Finally, the framework can support research, e.g. investigating under what situational or personal conditions the described competences of problem-solving can be taught. Here, future empirical work is needed that is based on a theoretical framework like ours.

## Theoretical background

In the following, we give a short introduction to problem-solving by focusing on the general structure of problem-solving processes. This will give us an overview of the important domain-independent approaches that reflect, for example, more general reasoning (Duncan, Chinn, and Barzilai 2017). Then, we summarize the prominent procedures of domain-specific problem-solving in the STEM domains because procedures in these fields vary, and we wish to incorporate the different characteristics of the domains into our framework. We focus on approaches, such as inquiry, that can be related to empirical work. Examples of this are hypothetico-deductive, hypothetico-inductive, and observational-inductive methods (see Mahootian and Eastman 2008). Such approaches will be denoted as *empirically orientated*. We also examine approaches that are solely based on logical inferences, such as proofs in mathematics (see Epp 1994); these are denoted as solely *theory-orientated*. The brief review leads to an overview of the problem-solving procedures used in STEM domains from which we derived our general epistemological framework.

## Structuring problem-solving

A complex problem can be defined as a non-routine situation in which a person tries to follow aims in a partially non-transparent, dynamic, and polytelic environment (Funke 2004, 291). Funke (2004) describes a set of assumptions that influence problem-solving, including that a person constructs a cognitive representation of the problem in a step-wise manner and that the problem-solving person seeks to use prior knowledge to achieve a solution. Both aspects illustrate the cognitive dimension of problem-solving in specific contexts or domains, which plays a central role in our work. Before we focus on those sources that are relevant for our work, we will give a definition of problem-solving. A widely accepted framework of problem-solving was introduced by the PISA-consortium:

Problem solving begins with recognising that a problem situation exists and establishing an understanding of the nature of the situation. It requires the solver to identify the specific problem(s) to be solved and to plan and carry out a solution, along with monitoring and evaluating progress throughout the activity. (OECD 2013, 123)

From a psychological point of view, problem-solving occurs to overcome barriers between a given state and a desired goal state by means of behavioural and/or cognitive multi-step activities (see Frensch and Funke 2005; Hussy [1998, 20]; Klieme,

Leutner, and Wirth 2005). Overcoming barriers is also an essential point in problem-solving within mathematics education (Schoenfeld 1985, 11; Vollrath 1992, 1). Furthermore, heuristic strategies are considered to be of special importance in mathematical problem-solving (Polya 1957, 52; Schoenfeld 1985, 69; Ewen 1996, 1). Priorities in problem-solving within natural science education are slightly different from those in mathematics; the importance of domain-specific knowledge is emphasized more in physics than in mathematics education. However, both heuristic strategies and domain-specific knowledge are essential for problem-solving in all domains. For a comparison of perspectives on problem-solving in mathematics, physics, and engineering education, see Lehmann (2018, 49).

Important for our argument, structuring helps students to acquire problem-solving skills (Booth et al. 2017; Williams and Otrell-Cass 2017; Greiff, Kretzschmar, and Leutner 2014). Structuring teaches students how to think by teaching them the general steps of problem-solving. Based on this assumption, numerous models have been developed that have different foci and serve different purposes. In the following, we briefly name a few to show the spectrum of approaches (see Table 1).

Merrill et al. (2017, 73) focus on the following five general steps: *identifying a problem, defining the problem, generating solutions, evaluating/choosing/enacting solutions, and assessing the outcome*. With respect to problem-solving in mathematics, the seminal contribution of Polya (1957) highlights four steps: *understanding the problem, devising a plan, carrying out the plan, and looking back*. Even though this model originates in mathematics, these general steps can be applied in other domains as well. Other frameworks have specific foci (e.g. Bransford and Stein 1984; Reif 1983; OECD 2013; Koppelt and Tiemann 2008). Oser and Baeriswyl's (2001) model, for example, also consists of five steps – in this case, *generating a problem, specifying a problem, finding possible ways to solve the problem, testing these ways, and putting the solution into a larger context* – but they put more emphasis on transfer activities that relate the solution of a specific problem to the students' prior knowledge. In contrast, Windschitl, Thompson, and Braaten (2008, 955) focus on inquiry by listing conversations that support model-based inquiry: *organizing what we know/would like to know, generating hypotheses, seeking evidence, constructing an argument, and setting broad parameters*. All of these steps, however, can be seen as stages of a general framework of problem-solving that proceed beyond inquiry work. Wu and Adams (2006) based their five dimensions of problem-solving – *reading/extracting all information from the question; real-life and common-sense approaches to solving problems; mathematics concepts, mathematization, and reasoning; standard computational skills; and carefulness in carrying out computations* – on their usefulness for teaching, the possibility to observe corresponding student activities, and the accessibility of students' results. Thus, their approach focusses on a measurable operationalisation of problem-solving.

Despite the different foci of problem-solving structures, Merrill et al. (2017) emphasize the general importance of sequencing instruction when teaching problem-solving. For our goal of developing an epistemological framework, this means that such a framework should be based on a well-justified structure. Furthermore, the brief review above shows the wide spectrum of approaches to problem-solving that should be reflected in our framework. In addition to a general view on the structure of problem-solving, our framework must also include methods and procedures of the different STEM domains. In doing so, we align with

**Table 1.** Different general models of problem-solving and their structure.

| Model   | Steps of problem-solving                             |   |  |  |  |
|---|--|---|--|--|--|
|   | Recognizing that a problem exists                    | Establishing an understanding of the nature of the situation, identifying the specific problem(s) | Planning a solution                                  | Carrying out a solution                | Monitoring and evaluating progress throughout the activity |
| PISA (OECD 2013)                              | Identifying a problem                                | Defining the problem  | Generating solutions                                 | Evaluating/choosing/enacting solutions | Assessing the outcome                                      |
| Merrill et al. (2017, 73)                     | Understanding the problem                            | Devising a plan   | Carrying out the plan                                | Looking back                           |  |
| Polya (1957)                                  | Generating a problem                                 | Specifying a problem  | Finding possible ways to solve the problem           | Testing these ways                     | Putting the solution into a larger context                 |
| Oser and Baeriswyl (2001)                     | Organizing what we know/would like to know           | Generating hypotheses   | Seeking evidence                                     | Constructing an argument               | Setting broad parameters                                   |
| Windschitl, Thompson, and Braaten (2008, 955) |  |   |  |  |  |
| Wu and Adams (2006)                           | Reading/extracting all information from the question | Real-life and common-sense approaches to solving problems   | Mathematics concepts, mathematisation, and reasoning | Standard computational skills          | Carefulness in carrying out computations                   |

Kind and Osborne (2017, 21), who emphasize the importance of domain-specific knowledge. The following paragraphs will analyse the different domains by distinguishing empirically and solely theory-orientated ways of problem-solving.

### **Empirically orientated ways of problem-solving**

Empirically orientated ways of problem-solving, similar to making inductive causal inferences (Kuhn et al. 1995) or following hypothetico-deductive paths (Lewis 1988), are based on generalizing evidence in the light of prior knowledge. The use of empirical data is prominent in the STEM domains. Thus, we will analyse these domains with the aim to identify methods and procedures of problem-solving that can inform our framework.

### **Science**

In science, problem-solving is often embedded in inquiry learning. So, inquiry can be seen as one way to solve problems. However, distinguishing between problem-based learning and inquiry learning is not easy. Hmelo-Silver, Duncan, and Clark (2007, 100) see the major distinction between the two only in their different origins (i.e. medical education for problem-based learning and science education for inquiry learning) and state that they see no clearly distinguishable features. Fortunate for our analyses, such a distinction is not important. Instead, it is important to identify ways in which inquiry processes are performed. Before we do this, we briefly quote our view of inquiry, as defined by the United States' National Research Council:

a multifaceted activity that involves making observations; posing questions; examining books and other sources of information to see what is already known; planning investigations; reviewing what is already known in light of experimental evidence; using tools to gather, analyse, and interpret data; proposing answers, explanations, and predictions; and communicating the results. (1996, 23)

Inquiry-based learning is a central approach in science education in which learners use, imitate, or simplify the methods and practices of scientists to 'rediscover' fundamental scientific principles and concepts, thereby, actively constructing their own knowledge (Van Joolingen et al. 2005). However, learners are not researchers, so there are, of course, differences between educational inquiry processes and authentic scientific research (Chinn and Malhotra 2002; Cobern et al. 2010).

When reviewing the literature on inquiry learning, it is difficult not to come across the term *scientific discovery learning*, an approach that puts emphasis on self-regulated and minimally or even unguided inquiry learning (Hmelo-Silver, Duncan, and Chinn 2007). The benefits and obstacles of scientific discovery learning, with respect to the amount guidance, are discussed elsewhere (Hmelo-Silver, Duncan, and Chinn 2007). Because scientific discovery learning can be seen as a problem-solving process (Van Joolingen and De Jong 1997, 308), we briefly summarize the well-known *scientific discovery as dual search* (SDDS) model (see Klahr and Dunbar 1988). The model differentiates between the *hypothesis space* (where a fully specified hypotheses is generated, e.g. that the mass of the bob of a simple pendulum does or does not have an influence on the time of oscillation) and the *experimentation space* (where an investigation is conducted that tests the hypothesis, e.g. times are measured using a pendulum with different masses), and it

relates both spaces through the process of *data evaluation* (where a justification is constructed that decides if the hypothesis was correct). Each space is related to a number of processes (Van Joolingen and De Jong 1997), and, thus, the *scientific discovery as dual search* model structures activities of experimentation. Other models of experimentation or laboratory work (Millar, Le Maréchal, and Tiberghien 1999; Osborne 2014; Basey and Francis 2011) describe comparable practices on the same medium-grained level. More detailed general paths of inquiry are described in a variety of other models (Pedaste et al. 2015; Riga et al. 2017; Cobern et al. 2010).

Thus, inquiry work was modelled for science and science education, and phases of scientific activities were identified that describe a circular process (see, for example, Wilhelm and Beishuizen 2003; Hartmann et al. 2015, 45]). In their review, Pedaste et al. (2015) specify these phases for educational inquiry processes and show that all frameworks follow a set of similar phases of scientific activities within a circular process even though the order of the phases varies. Three main approaches are suggested by the authors, the steps of which are detailed below (Pedaste et al. 2015, 54, 56):

- (a.) Orientation (introduction of a learning topic by the environment or teacher, identifying the main variables, stating a problem, and stimulating curiosity); Questioning (generating research questions or more open questions based on theory); Exploration (systematic and planned data generation, carrying out an investigation with the intention of finding a relation among the variables involved); Questioning, Exploration, and Data Interpretation (making meaning of collected data and synthesis of new knowledge, formulating the relations between variables); and Conclusion (comparing inferences based on data with hypotheses or research questions).
- (b.) Orientation and Hypothesis Generation (arriving at a testable hypothesis based on theory, making a statement); Experimentation (designing and conducting an experiment, making and applying a strategic plan for the experiment with a specific timeline, collecting evidence for testing a hypothesis); Data Interpretation and Hypothesis Generation (on the basis of research questions); Experimentation, Data Interpretation, and Conclusion.
- (c.) Orientation, Questioning, Hypothesis Generation, Experimentation, Data Interpretation, (Questioning) Hypothesis Generation, Experimentation, Data Interpretation, and Conclusion.

These three approaches differ in the conceptualization of the problem and in the type of investigation. The authors recommend that 'If the students have no specific idea and only a general plan of what to explore, they should start from more open question(s) that guide them' (Pedaste et al. 2015, 56), which correlates to approach (a.). Corresponding with (b.), 'If the students have a more specific, often theory-based idea about what to investigate, then a hypothesis-driven approach is suitable' (Pedaste et al. 2015, 56). Finally, there is a 'question-driven approach, where students have a question and their next goal is to collect background information for stating a specific hypothesis as a possible answer to the question' (Pedaste et al. 2015, 56), or approach (c.).

The brief review shows that steps like *data interpretation* or *conclusion* are of general importance, for example, when judging the quality of measurement data (Priemer and



Hellwig 2018), describing the conceptualization of evidence (Duncan, Chinn, and Barzilai 2017), or reasoning on the basis of evidence (Osborne, Erduran, and Simon 2004). Reasoning is especially not limited to empirical inquiry but is general to problem-solving and scientific work. Hence, we close this section by briefly describing two frameworks of reasoning in science.

A model of scientific reasoning that has received a lot of attention is that described by Osborne (2013). He details three principal spheres of activities: *investigating (experimentation)*, *developing explanations and solutions (hypothesis generation)*, and *evaluating (evidence evaluation)*. This model reflects different forms of reasoning by both showing how these activities are interrelated and what form of reasoning focusses on which sphere. Thus, the model is a very useful framework in science education. However, since it offers a 'simplified model of the major spheres within which scientific reasoning takes place and its major function' (Osborne 2013, 271), the activities described are not outlined in detail.

In a more recent work, Kind and Osborne (2017, 11) describe six styles of scientific reasoning: *mathematical deduction*, or 'the use of mathematics to represent the world and for deductive argument'; *experimental evaluation*, or 'the use of empirical investigation to establish patterns, differentiate one form of object from another, and to test the predictions of hypothetical models'; *hypothetical modelling*, or 'the construction of analogical and hypothetical models to represent the world'; *categorization and classification*, or 'the ordering of variety by comparison and taxonomy'; *probabilistic reasoning*, or 'the statistical analysis of regularities in populations, the identification of patterns, and the calculus of their probability'; and *historical-based evolutionary reasoning*, or 'the construction of historical accounts of the derivation of the development of species, the Earth, the solar system the universe, the elements and more'. The authors emphasize that teaching in science – the styles focus on science without explicitly addressing engineering, computer science, or mathematics – should cover all of these styles if they are to reflect knowledge acquisition processes as well as ontological, methodological, and epistemic diversity (Kind and Osborne 2017, 16). The epistemic entities of the different styles, such as deduction, hypothesis, experimental test, prediction, categorizing, and probability, are especially important for our work (Kind and Osborne 2017, 14) because they shape the multiple methods and processes of problem-solving. It is this wide spectrum of methods that brings science forward and makes it so successful (Brandon 1994).

Hence, our framework must reflect this multiplicity by showing, for example, that investigations in problem-solving processes can lay somewhere in between: the 'space of experimentality' spanned by the two dimensions; *manipulation*, in which the experimenter manipulates nature in some way versus nature being solely observed as it is; and hypothesis testing, in which a hypothesis is tested or not (Brandon 1994, 66). This means, for example, that 'not all experiments involve hypothesis testing and that not all descriptive work is non-manipulative' (Erduran and Dagher 2014, 100).

The wide spectrum of cited work on problem-solving in science education provides multiple methods and processes. Before we describe how these informed our framework, we will summarize empirically orientated work in mathematics, engineering, and computer science education, and we will conclude by highlighting the relevance of solely theory-orientated ways of problem-solving. As can be seen below, the corpus of literature on empirically orientated work is considerably smaller in mathematics, technology, engineering, and computer science education compared to in science

education. This is because empirically orientated work has a shorter tradition in the education of these domains.

### Mathematics

Problem-solving processes in mathematics were modelled on a set of steps and heuristic strategies originated by Polya (1957). As shown in the now-classic educational film 'Let Us Teach Guessing', Polya (1966) refers to a *hypothesis* as a 'reasonable guess'. Reasonable guesses can be verified or refined and generalized but also rejected by observing different cases or examining in the light of empirical data. Even though this approach goes back to the 1950s and 1960s, *inquiry-based mathematics education* (IBME; see Artigue and Blomhøj 2013; Maaß and Artigue 2013; Maaß and Doormann 2013; Engeln, Euler, and Maaß 2013) and *mathematical experimentation* (Leuders, Naccarella, and Philipp 2011; Leuders and Philipp 2014) are current approaches that have received increasing attention only recently. Artigue and Baptist (2012, 8) list several aspects of mathematical inquiry processes, including: the *role of exploration* (familiarity with the problem increases); the *non-linearity of the process* (multiple steps in varying order may lead to a solution); the *definitive nature of the results* (no further experience will invalidate the results); and the *importance of finding generalizations* (and how these contribute to a new understanding). Even though the wording is different, these aspects resemble the activities in scientific inquiry.

However, there are differences between mathematics and science. Mathematics attempts to finalize problems through deductive steps like proofing (Leuders and Philipp 2014, 185), which can be different when 'regarding the type of questions it addresses and the processes it relies on to answer them' (Artigue and Baptist 2012, 4). For example, mathematics creates its own objects and realities that do not necessarily relate to entities of the natural, social, or cultural world. Hence, corresponding problems can arise from internal questions, such as 'What is the greatest product that can be obtained by decomposing a positive integer into a sum of positive integers and multiplying the terms of the sum?' (Artigue and Baptist 2012, 5). Solving problems like this one does not necessarily require empirical data; however, numbers can be used to explore the problem in testing processes or when working with special cases (Leuders and Philipp 2014). Numerical or computer-assisted proofs in mathematics are another example in which data sets are used to solve problems. Thus, problem-solving in mathematics can utilize features similar to inquiry in science:

Like in natural sciences, we may have hands-on experiments (e.g. paper knotting and paper folding, devices like a pantograph, games like tangram or tower of Hanoi), we have thought experiments (very typical for maths), and we have experiments on a computer screen with the help of appropriate software (dynamic worksheets). (Artigue and Baptist 2012, 15)

This shows that mathematics does use empirically orientated ways of solving problems.

### Engineering and technology

Even though engineering and technology are not the same – engineering can be seen as an application process of science in which technology is a result – we discuss both fields together because we found no profound differences concerning problem-solving activities in education.

There are models in engineering that clearly relate to Polya's (1957) problem-solving phases for mathematics. Gray, Constanzo, and Plesha (2005), for instance, suggest a five-stage linear model, including the stages: *road map*, *modelling*, *governing equations*, *computation*, and *discussion and verification*. Woods (2000), however, suggests a cyclic model that encompasses six stages. Basing his model on a broad literature review, he calls his stages: *engage: I want to, and I can; define the stated problem; explore; plan; do it; and look back*. These two examples show that there are strong similarities between scientific and engineering problem-solving. For example, *posing questions*, *developing empirical tests*, and *drawing conclusions* are inquiry steps in both science and engineering. Marulcu and Barnett (2016, 87) summarize the impact of design activities in science learning and point out that engineering design deals with real problems of societal and personal relevance, offers ways to understand the nature of scientific and technological knowledge acquisition, motivates students to study science, and shows that problems may have multiple representations and solutions. All of these aspects reflect scientific work.

However, engineering problems with a design-based approach also often 'begin by posing a design challenge, which limits the ability for students to begin with asking self-motivated questions worthy of scientific investigation' (Mehalik, Doppelt, and Schuun 2008, 73). Mehalik, Doppelt, and Schuun (2008, 74) describe seven steps of a *systems design approach*: *describe the current situation*, *identify needs*, *develop criteria*, *generate alternatives*, *choose an alternative*, *create a prototype/test*, and *reflect and evaluate* (for a slightly different model, see Marulcu and Barnett 2016, 90). This approach focuses more on the development and optimization of a process or device and much less on the identification of basic principles or concepts, as is the case in natural sciences and mathematics. This focus on design and artefact creation is a typical difference between the empirical work in science and mathematics, on the one hand, and engineering and parts of computer science on the other. Kolodner, Gray, and Fasse (2003) unite the scientific views with the engineering by defining two parallel cycles of activities in design-based learning: the *design/redesign cycle* and the *investigate and explore cycle*. Design and redesign focus on problem-specific activities that include processes like understanding the special needs, planning a design, presenting and sharing results, constructing and testing a prototype, analysing and explaining its performance, and presenting and sharing the results. The investigation and exploration cycle describes a more general approach to problem-solving, including steps like *clarify question*, *make hypothesis*, *design investigation*, *conduct investigation*, *analyse results*, and *present and share*. Marulcu and Barnett (2016) summarize Kolodner, Gray, and Fasse (2003) work as follows:

The sequence of the design/redesign cycle includes playing with materials and devices to understand the challenge; engaging in problem-based learning to define what needs to be investigated; planning a design; and constructing, testing, and analysing the design. The sequence of the investigate and explore cycle includes clarifying the question, generating a hypothesis about it, designing the investigation, conducting and analysing, and finally presenting and sharing it in a poster session. (Marulcu and Barnett 2016, 89)

However, the two cycles empirically appear to be separated, and their interplay is not obvious. Nonetheless, we can conclude that based processes are used in problem-solving activities in engineering and technology education.

### Computer science

There are some studies that report the use of problem-solving approaches in computer science (see, for example, Kay et al. 2000; Pucher and Lehner 2011). However, little information is provided concerning the steps implemented in the learning instructions. Usually, the authors refer to a general structure of problem-solving, such as activating prior knowledge, connecting the learning to specific problem situations, and making the students elaborate on the material that they have learned (Nuutila, Törmä, and Malmi 2005, 124). A more detailed seven step structuring of problem-solving in computer science is provided by Nuutila, Törmä, and Malmi (2005, 127). They differentiate between: *examination* (the group gets familiar with the case material); *identification of the problem* (an initial title for the case is specified); *brainstorming* (associations are presented); *sketching of an explanatory model* (an initial version is constructed); *establishing the learning goals* (identifying central questions); *independent studying* (individual work on questions); and *discussion about learned material* (explanation of the central concepts, analysis of the material, and evaluation of the solution). As it was described for mathematics, the solutions to the problems can (but also do not have to) include empirical data.

Some areas of computer science are experimental by nature, though. An example that is currently aggregating much attention is *physical computing* (Przybylla and Romeike 2015), which is strongly related to science and engineering inquiry due to the use of programmable sensor-equipped devices, such as robots, that gather data from an environment, store them, and/or react with certain programmed routines (Schulz and Pinkwart 2016). On the one hand, these routines can be optimized to gain the best performance from a device, a process that resembles inquiry in engineering. Thus, computer science uses empirically orientated approaches to problem-solving that are frequently similar to those in technology and engineering since the goal of solving a problem involves the design and creation of a software or hardware device. On the other hand, routines can provide the empirical data required to discover scientific principles, which resembles inquiry in science. Furthermore, with the recent advent of data analytics in education as well (e.g. AI based data analysis methods), one can argue that computer science education is increasingly emphasizing the scientific view of inquiry.

Another role that computer science often plays in problem-solving is that of enabling technology; problem-solving approaches have been implemented in many computer-supported learning environments (De Jong et al. 2010; Mäeots, Pedaste, and Sarapuu 2011); however, the content addressed in these environments originates from the natural sciences and not from computer science.

### Solely theory-orientated paths of knowledge acquisition

Besides empirically orientated paths of knowledge acquisition, which are based on empirical work followed by generalizations, there are other deductive ways to gain knowledge. Proofs in mathematics and derivations in physics are methods that use axioms or basic definitions, which are taken to be valid, to arrive at logical conclusions and inferences. In mathematics education, general deductive paths of problem-solving (Neumann et al. 2015; Polya 1957; Schoenfeld 1985); more specifically, proofs have been

investigated for a long time (for geometry, see Beckmann 1989; Reiss and Heinze 2004). Proofing is a complex endeavour; therefore, to inform teaching and learning processes, Boero (1999) developed a simplified process model for proofing, which is comprised of six steps: *production of a conjecture*; *formulation of the statement according to shared textual conventions*; *exploration of the content of the conjecture*; *selection and enchaining of coherent, theoretical arguments into a deductive chain*; *organization of the enchainment of arguments into a proof*; and *approaching a formal proof*. In the model, the proofing process and its different stages come to the forefront and can be the subject of classroom discourse on different levels. As can be drawn from this process model, inductive, explorative, and deductive moves are inherent in any process model of proofing.

However, proofing is not the only solely theory-orientated way of problem-solving. Observed from a more general perspective, inquiry can be purely theoretical as well. Artigue and Baptist (2012, 15) list several practices of inquiry-based mathematics, such as questioning, exploring, observing, discovering, assuming, explaining, and proving, that need not necessarily be based on empirical data. This underlines the general importance of solely theory-orientated ways of problem-solving. It is interesting to note that *exploring* can be described as conducting a *Baconian Experiment* (Leuders and Philipp 2014, 178) in which knowledge acquisition is performed in an abductive way. In contrast, *Kantian Experiments* (Leuders and Philipp 2014, 178), like thought experiments, are deductive by nature. Based on this, Leuders and Philipp (2014, 180) list nine steps of experimentation in mathematics: (1) focusing on the relevant aspects of a problem, collecting appropriate tools; (2) checking existing theories regarding how much of the problem is already explained; (3) exploring the problem by systematically varying the phenomenon; (4) constructing hypotheses; (5) conducting the experiment; (6) checking the conformity of the results with the hypotheses; (7) analysing the results, judging how plausible the hypotheses are; (8) relating the results to existing theories, deriving the results through deductive ways; and (9) publicizing and discussing the results. This model shows that experimentation in mathematics can generate hypotheses as well as have a confirmative character. Again, these steps can but may not be based on empirical data.

Finally, modelling processes can be performed solely based on theory as well. The modelling circle that Artigue and Baptist (2012, 15) describe utilizes the following steps: *formulation of a task* (constructing the object of the modelling process); *selection and construction of the relevant objects* (making the mathematical representation possible); *transformation and translation of selected objects* (abstraction and idealization); *using mathematical methods* (achieving mathematical results); *interpretation* (drawing conclusions); and *evaluating the validity of the model*. In the last step, the model is evaluated by 'comparison with data (observed or predicted) and/or with already established knowledge (theoretically based or shared/personal experience based)' (Artigue and Baptist 2012, 15). Here, it becomes clear that data can but does not have to play a role in modelling processes when solving problems. This conclusion, that inquiry and modelling can be solely theory-orientated as well, is true for all STEM domains because none of the steps cited above or activities are inherent to mathematics alone.

Our brief discussion of problem-solving shows that the STEM domains share common processes (e.g. exploring) while having specific characteristics (e.g. prototyping). The

distinctions between the activities of scientists in differing fields are discussed very clearly by Hodson and Wong (2017, 9). Against the background, that there is an increasing demand for integrated STEM education when working on natural phenomena, authentic problems (Krajcik 2015), or discussing the nature of science (Dagher and Erduran 2017; Hodson and Wong 2017), a general model is needed. Consequently, we developed a comprehensive STEM framework of problem-solving that structures and interweaves the methods and processes of mathematics, computer science, natural sciences, technology, and engineering. In the following section, we explain how our framework was developed.

## Constructing a framework of problem-solving in the STEM domains

The brief review of how problem-solving is structured in different domains describes the sources we used to collect, compare, and extract the methods and processes of problem-solving used in the STEM domains at a fine-grained level. Our aim was the construction of an integrated framework of problem-solving that could be used for domain-specific and, most importantly, interdisciplinary problems. That meant that the framework should reflect the multifaceted ways in which scientific problems can be solved by providing a set of very different steps of processes and methods to choose from. Thus, the framework offers a whole spectrum of approaches. For the development of the framework we used the following method.

### Method used to develop the framework

We developed the model by following four steps. First, we choose the PISA model (OECD 2013) as a general model of problem-solving to acquire a structure that helps to organize the steps of problem-solving. This very general model has the advantage that it can be applied to all STEM domains without having a special focus (see Table 1). However, other models (for example, Merrill et al.'s 2017, 73]) could have served the same purpose as well. Second, we went through all domain-specific models (as described in the theoretical background) and assigned the steps of problem-solving to the general structure. Third, we eliminated duplications and synonyms that appeared in the different models, thus, arriving at a framework that lists several different steps that can be assigned to the general structure (see Table 2). This first representation of our framework showed the multiplicity of problem-solving activities. However, Table 2 does not show how the steps are related to each other, so the sequences of steps that describe the processes of problem-solving cannot be represented in Table 2. For that reason, in the fourth step, we ordered the steps in a flowchart (see Figure 1) so that the problem-solving activities could be shown as processes. Here, we included the complete processes of problem-solving that consist of multiple steps found in the literature.

In Figure 1, the framework is visualized with arrows between the different steps, which are represented by boxes. Each step/box contains different but comparable activities that belong to the step, for example, specifying a problem, identifying needs, asking a question, or stating a hypothesis or conjecture. These activities can be alternatives, or more than one activity can be relevant for solving a certain problem. We added the step *communicating results and methods* to the framework even though it is

**Table 2.** Relations between a general structure of problem-solving and the steps of our STEM and computer science framework for problem-solving.

| General structure of problem-solving (OECD 2013)   | Steps of the STEM and computer science framework of problem-solving   |
|--|---|
| Recognizing that a problem situation exists  | 1. Noticing/generating a problem, phenomenon, observation, example, exception, irregularity, or situation   |
| Establishing an understanding of the nature of the situation, identifying the specific problem(s) to be solved | 2. Exploring, examining, trial-and-error, tinkering, optimizing, guessing, or discovering   |
| Planning a solution  | 3. Specifying a problem, identifying needs, asking a question, stating a hypothesis or conjecture   |
|  | 4. Making predictions, developing criteria, assuming relations, or checking existing theories   |
|  | 5. Developing explanations and solutions, generating and choosing alternatives, planning a proof, design, test or experiment, building models, collecting tools |
| Carrying out a solution  | 6. Deducting, proofing, or finding counter-examples   |
|  | 7. Conducting a test or experiment, creating a prototype, using models, testing solutions, or computing   |
|  | 8. Generating data or observing   |
|  | 9. Manipulating, organizing, and representing data, categorizing observations, or choosing an alternative   |
| Monitoring and evaluating progress throughout the activity   | 10. Evaluating data and observations or making validations  |
|  | 11. Rejecting or supporting and refuting or accepting a hypothesis/solution   |
|  | 12. Making inferences, interpreting, reflecting on results and methods, embedding in contexts, or making conclusions  |
|  | 13. Communicating results and methods   |



not necessarily a step of problem-solving because it is a common practice to share the results derived from this process, and it is an activity of high importance in education. Thus, Figure 1 shows the multiple processes using which problems can be solved.

In the following, we will show that the models we cited in the theoretical background are represented in our framework. We do this exemplarily by choosing one model for each of the STEM domains. Pedaste et al.'s (2015) model (a.) of inquiry in science contains the steps: *orientation, questioning, exploration, questioning, exploration, data interpretation, and conclusion*. These steps correspond to the following sequence of steps in our model (see Figure 1): 1., 3., 2., 3., 2., and 12. In mathematics, Boero's (1999) model utilizes the steps: *production of a conjecture; formulation of the statement according to shared textual conventions; exploration of the content of the conjecture; selection and enchaining of coherent, theoretical arguments into a deductive chain; organization of the enchain arguments into a proof; and approaching a formal proof*. These correspond to the steps 3., 2., 3., 4., 5., and 6. in our model. Following, Mehalik, Doppelt, and Schuun (2008) model, as an example for engineering and technology, distinguishes between: *describe the current situation, identify needs, develop criteria, generate alternatives, choose an alternative, create prototype/test, and reflect and evaluate*. The corresponding steps in our framework are 1., 3., 4., 5., 7., 10., and 12. Finally, Nuutila, Törmä, and Malmi (2005) model for computer science follows these steps: *examination, identification of the problem, brainstorming, sketching of an explanatory model, establishing the learning goals, independent studying, and discussion about learned material*. Even though this model focusses on leaning sequences and methods, we can assign the activities to the following steps in our model: 1., 3., 4., 5., 7., 8., 10., and 12.

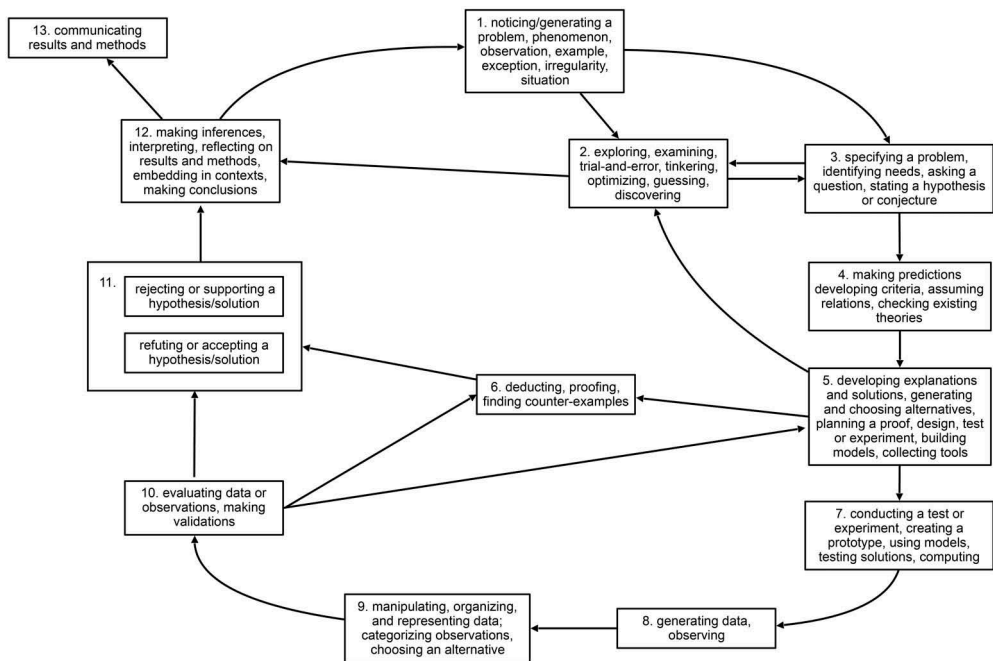


Figure 1. Visual representation of an integrated framework of problem-solving.



The representation of the framework in [Figure 1](#) has limitations. Even though the arrows indicate a logical main direction for problem-solving activities, steps may be pursued in the opposite order, and steps may be repeated or omitted. Further, there may be connections between steps that are not represented by arrows. Finally, [Figure 1](#) contains multiple cycles and steps that reflect many different ways of problem-solving but do not necessarily mirror the practices of all scientists or all possible learning sequences (see also Hartmann et al. 2015). In this sense, the model describes the idealized logic of different approaches to problem-solving but should not be read as a description of the standard routes that scientific work always follows in the STEM domains or in STEM education.

The framework represents processes of problem-solving for the involved domains. In the following, we illustrate the framework by describing example paths of problem-solving. We do so by giving one example that reflects an authentic interdisciplinary problem and, further, by describing four paths that exemplify solely theory-orientated, optimization-orientated, exploration-orientated, and empirically orientated approaches.

### ***Examples that illustrate the use of the framework***

#### ***Path 1***

Imagine that a class in a high school wants to find out whether the noise on the street where the school is located is unhealthy for the students. This describes a general and authentic problem situation (step 1). Let us assume in our example that the students have enough information and prior knowledge to specify the problem (step 3), e.g. by asking: ‘Does the sound pollution of the street excel a certain threshold?’ The next step of the students could be that they begin to explore how sound levels are measured and how the conditions in which they want to answer their question can be specified in more detail (step 2). The result can be a more precise question (step 3): ‘Does the maximal sound pollution of the street excel a certain threshold when measured in a classroom with windows closed?’ After that, the students set a threshold value for the sound level (step 4) and plan an experiment (step 5) by choosing measurement devices, choosing rooms facing the street in different levels of the building, and determining the number of measurements they need. These activities are followed by conducting the experiment (step 7), generating data (step 8), organizing the data (step 9), and evaluating the data (step 10). In the last step, let us assume that the students notice that to find out if the street is the only source of sound pollution, they need reference measurements. So, they modify their experimental plan by adding measurements in rooms of the building that do not face the street (step 5). Again, they follow steps 7, 8, 9, and 10. The result could be that the students find out that the street is a relevant source of sound pollution in certain rooms of the building (step 11). Thus, they conclude that these rooms should be equipped with sound insulated windows (step 12) and communicate this to the head of the school (step 13).

This example illustrates one possible path through the framework to solve a problem. The benefit of the framework is that it offers a variety of different activities that help to solve a problem. However, the framework does not solve the specific problem for the students. Nonetheless, given that the students understand the steps, the framework can

serve as a tool box that offers options and helps them not to forget important processes. In order to illustrate the use of the framework, we outline four additional paths through the framework.

### Path 2

Problem-solving with proofs or derivations in mathematics, theoretical physics, or theoretical computer science is included in the framework and is represented with the steps: *generating a problem, specifying a problem, stating a hypothesis, checking existing theories, planning a proof, proofing, refuting or accepting a hypothesis, and making inferences and embedding in contexts*. Here, empirical work, which is often a core practice in science and engineering and is always subject to constraints like uncertainties in data, is not involved. Thus, this solely theory-orientated approach includes mathematical deduction and hypothetical modelling and may, in some cases, include historical-based evolutionary reasoning (using the notion of Kind and Osborne 2017, 11]). Examples include the proof of the Pythagorean theorem (mathematics), the deviation of Archimedes' principle with the definition of pressure and hydrostatic pressure (physics), and the estimation of a population size with exponential functions (biology).

### Path 3

In engineering and practical areas of computer science, the optimization of a device or effect can be the goal of development. This is embedded in the model through the steps: *noticing a problem; identifying needs; developing criteria; developing solutions; creating a prototype; generating data*, which includes making experiences with the prototype; *evaluating; rejecting or supporting the solution and making inferences; embedding in contexts*; and *reflecting results and methods*, which may lead to modifying a prototype. Examples include optimizations in experimental settings, such as arrays of optical instruments (physics), empirically based manipulations of aircraft wings in wind tunnels (engineering), and simulations of bacteria growth (biology). Examples in computer science include the design and improvement of robotics devices that are supposed to exhibit a certain complex behaviour, which is typically addressed in an iterative manner that gradually improves the solution.

### Path 4

*Exploring* describes a process in which a problem or phenomenon is investigated without necessarily having a theory or hypothesis at hand (see Steinle 2002). Characteristics and regularities of a phenomenon, problem, or observation are investigated by *examining, trying things out, tinkering, or guessing* – for example, by running experiments first (physics, chemistry, biology); investigating special cases (physics, mathematics); testing known prototypes (engineering); or working on the problem using strategies or algorithms (mathematics, computer science). The results of an exploration can include categorizations and classifications as well as identifications of regularities and patterns with corresponding probabilities (see Kind and Osborne 2017, 11]). Furthermore, the result of an exploration can be a new phenomenon/problem, the specification of a known phenomenon/problem, or – if more information is available – a first hypothesis. Examples include data selection in particle physics (Karaca 2017); testing in how many

parts 1, 2, 3, ... (hyper-) planes divide the 3- (n-) dimensional space (Polya 1966); and the observation of the behaviour of animals under certain conditions (biology).

### Path 5

Finally, the 'typical' path of problem-solving in science through inquiry is embedded in the model in the steps: *noticing a phenomenon; asking a question; stating a hypothesis; planning an experiment; conducting an experiment; generating, manipulating, and evaluating data; rejecting or supporting the hypothesis; and making inferences, embedding in contexts, and reflecting on results and methods*. Kind and Osborne (2017, 11) describe this as experimental evaluation. Historical-based evolutionary reasoning may also fall under this approach if the researcher strives to find empirical data that supports the underlying hypotheses. Examples include Newton's law of cooling in physics or the search for the missing links in evolution. Similar procedures are used in mathematics when different cases are explored by calculating examples or when computer experiments are conducted utilizing dynamic geometry software.

## Discussion

Problem-solving is a general teaching approach that can be applied in different domains. The corresponding steps and general processes were described in models that refer to single (e.g. mathematics) or a set of (e.g. science) domains. However, there is no integrated framework that covers all STEM domains and, therefore, no framework that could cover, for example, all of the five paths of problem-solving illustrated above. We posit that such a framework is necessary to solve authentic problems that address different domains simultaneously. Here, domain-specific models fall short. Against this background, we developed such an integrated STEM framework for problem-solving, but what benefits do we get from this model? We discuss this question with respect to research and practice below.

### The framework in research

Before we put our framework into the context of existing work in the field, we desire to outline that we can retrace our own steps of constructing the framework using the framework as well. As presented in the introduction, we noticed a problem with students having to be prepared to solve problems they will face in the twenty-first-century (step 1:); we specified the problem by showing the need for an integrated framework (step 3); then, we checked existing theories by collecting information about problem-solving activities in the STEM domains (step 4); after, we developed solutions, i.e. the construction of our framework (step 5); we deduced additional implications and solved inconsistencies and, thus, made the framework coherent (step 6); we supported a solution by declaring that the framework was a solution to the problem (step 11); we made inferences by discussing the benefits and the limitations of such a model (step 12); and, finally, we communicated the framework by publishing our work (step 13).

It has been repeatedly claimed that science education needs to reflect the multiple paths of knowledge acquisition and problem-solving that are used in science, for example, in the STEM domains (Kind and Osborne 2017; Erduran and Dagher 2014);

there is no such process as ‘the scientific method’ and no single form of reasoning. Furthermore, Kind and Osborne (2017, 25) argue that mathematical and computational thinking cannot be excised. All of this necessitates an integrated framework of problem-solving that holds for different domains. Clearly, there is a variety of models of problem-solving that denote general steps as described in the theoretical section. However, these models do not show the many different processes and steps of problem-solving in detail that originate from the different domains.

In contrast, our framework includes steps and approaches identified in STEM and, thus, reflects the multiplicity of the practices and activities that remain hidden in general models of problem-solving. We created our framework by reviewing work on problem-solving and knowledge acquisition in the STEM domains. We included all steps and paths of problem-solving that were found in the literature, so, it is safe to say that even though the framework may not be complete, it was deduced from multiple sources across all of the STEM domains. What distinguishes our framework from other work in the field is that: 1) it covers characteristics and details of problem-solving from all STEM domains (and, thus, is domain-specific and interdisciplinary at the same time); 2) it focusses on the steps and processes of problem-solving at a fine-grained level and, thus, differs from the more general styles of scientific reasoning (Kind and Osborne 2017) or the general models of problem solving; and 3) it displays a variety of different options for problem-solving without highlighting a single path as the correct or best practice.

The framework does not model the competences of problem solving; however, it can inform such models by highlighting which competences are important for solving a problem, for example: to know that there are different ways to solve scientific problems; to decide whether there is a need for an exploration; to choose the processes appropriate for the question at hand; to follow the indicated steps of activities and not omit steps or intentionally omit steps; to work with different problem-solving processes at the same time (e.g. explorations and deductions); to realize restrictions of the gained solution; and to know if a hypothesis or question can be answered empirically. All of these competences are linked to steps of the framework.

In doing this, the framework can help to explain the research results that report students’ difficulties with solving problems. For example, research has demonstrated that students show very limited abilities in constructing hypotheses (Walpuski and Schulz 2011), that students ‘fail to design rigorous investigations to generate evidence’ (Keys and Bryan 2001, 640), and that students ‘have difficulties conducting systematic scientific investigations’ (Edelson, Gordin, and Pea 1999, 399). In light of science inquiry models, these results indicate students’ weak competencies and inappropriate practical work because the observed activities do not match the models. However, a comprehensive STEM framework may identify these activities as procedures that are adequate in other domains. If, for example, students do not conduct systematic experiments by controlling variables but try to solve problems by using numerous examples or constructing a prototype (Schauble 1990), they apply processes often used in engineering. However, this does not necessarily mean that students’ methods are appropriate in general.

Clearly, there are scientific problems that cannot be solved by optimizing prototypes or phenomena. However, the framework identifies and classifies these student activities

and can help to compare and evaluate these approaches by showing the relations or alternatives to other activities in the framework, a view of the multiple ways that students should acquire (see Osborne 2013). So, in research, the framework can be used as a common reference for problem-solving processes that involve different domains. This may help to answer research questions like: can students who were taught activities to develop a proof in mathematics transfer these activities to deductive methods in physics?; can students use examples and special cases or empirical data (e.g. from numerical algorithms) like they are commonly used in science as well in mathematics to fruitfully construct a proof?; and can students see the similarities and differences between the variants of inquiry in science and in engineering? Here, the integrated framework serves as a reference, using which students' activities in the different domains can be compared, e.g. by showing the commonalities and diversities of the different domains. Two independent domain-specific models cannot provide a common standard for such a comparison.

There are open questions related to our framework. As made clear in the methods section, the framework is solely based on theoretical work that we originated from our review. Consequently, a next step would be to test the framework empirically. That means, for example, to investigate whether – and if so, under which conditions – the framework fosters students' competences in solving authentic and interdisciplinary problems.

### *The framework in practice*

Our interdisciplinary STEM framework can, unlike a discipline-specific model, help to construct learning environments that foster authentic problem-solving because it shows a wide spectrum of steps and processes inherent to problem-solving without restricting the problem to a certain domain from the outset. An understanding of the framework – for example, in its representation in Figure 1 – provides students with multiple options of possible activities to solve a problem. Thus, it opens the perspective of problem-solving to processes that may seem unfamiliar, e.g. to use prototyping to solve mathematical problems. So, the framework can help students to plan, monitor, and reflect on their own problem-solving processes. For example, if students try to solve a science problem by constructing a prototype or by using a single example as a general solution, the framework can inform them that an appropriate step in solving the problem is conducting an experiment that generates more general evidence. If students know the framework, and if they can locate their activity within the framework for the specific context they work in, they can discuss if their activity is appropriate for their given problem or task. We see the framework as a tool box that provides options for students to choose from in problem-solving activities. However, no solutions are given, and no hints are provided as to whether the step taken is appropriate.

Furthermore, the framework can assist by pointing out the differences and similarities between the STEM domains in discipline-specific and interdisciplinary learning environments. On the one hand, physics uses solely theory-orientated methods that resemble mathematical proofs. On the other hand, mathematics uses numerical methods like 'experiments' (e.g. dynamic geometry environments) that resemble the empirically orientated approaches commonly used in physics. Here, the differences between empirically orientated methods, which always come with constraints regarding the

generalizability of the results, and solely theory-orientated methods become apparent to the students. Such connections can only be demonstrated by a framework that spans across the range of the STEM domains.

With our framework, teachers have an easily accessible, visual representation of problem-solving processes – a support that addresses a problem identified by Krämer, Nessler, and Schlüter (2015, 336), who stated that in their study 46% of the participating teacher students had problems visualizing the inquiry processes. Thus, the framework may help them understand what it means to teach science through inquiry in schools, a prerequisite for the implementation of inquiry in teaching (Smit et al. 2017, 479). We suggest using the framework for teaching purposes in a way that is acutely linked to meaningful problems (as described by Krajcik 2015) and is applied to content (Kind and Osborne 2017). In this way, the framework can help students gain a comprehensive view of problem-solving methods and techniques used in the STEM domains, as recommended, for example, by Edelson, Gordin, and Pea (1999). Furthermore, the framework can help to reflect problem-solving processes after a problem is solved or after students have given up. All steps taken can be identified, retraced, and made visible in a representation like Figure 1. This helps to focus on scientific strategies of problem-solving, putting the solution into a larger context, and to relate the solution to the students' prior knowledge, which is a prerequisite to achieve competences in solving problems (Oser and Baeriswyl 2001).

We know that by providing our framework as a tool to solve problems that many complications emerge that we do not answer here. For example, we give no information on how to teach the framework, how to embed the framework into school curriculum, which authentic problems can be solved with the framework, how long it takes students to grasp the idea of the framework, or how students can make the best use of the framework. So far, we have to leave these questions open as the purpose of this paper is to discuss our framework in the STEM education community. Empirical investigations regarding the questions above are subject to ongoing research.

## Conclusions

We have developed a framework for problem-solving processes in the STEM domains based on a review of work in these fields. Such an integrated framework that covers all STEM domains at a fine-grained level is new. We believe that it is useful for research and practice because it provides an interdisciplinary view to foster students' problem-solving competences. These competences – for example, to identify problems, to review related information, to develop and evaluate options, and to implement solutions – are often seen and described as important twenty-first-century skills (Jang 2016, 298). Corresponding activities are described in our framework. However, the framework itself does not solve students' problems, but it does enable students to acquire the necessary competences to solve authentic problems by providing options of activities to choose from. Thus, we have provided a base to describe learning progressions and performance expectations regarding problem-solving.

Thus far, the framework is a theoretical construction. Teaching practice and empirical research are needed to show if and in which ways the framework is helpful. However, we do believe that such a framework is necessary to explicate adequately how science works and the multiple ways in which authentic interdisciplinary problems are solved.

## Note

1. Throughout this paper, we refer to science, technology, engineering, mathematics, and computer science but use the acronym STEM for the sake of convenience.

## Disclosure statement

No potential conflict of interest was reported by the authors.

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